

# Vector-like Fermions in a Minimal Composite Higgs Model

Haiying Cai

*Department of Physics, Peking University, Beijing 100871, China*

## Abstract

We consider the scenario where the composite Higgs arising as a pNGB in a two-site model with a non-local term included. Constraints from pion scattering and electroweak precision test are considered. We discuss the effects of composite resonances, in particular the one from composite vector-like fermions, on the oblique parameters. It is noticed that the gluon fusion production of Higgs boson is suppressed with respect to the Standard Model for about 6% after imposing the unitarity and electroweak bounds.

# 1 Introduction

With the discovery of the 125 GeV Higgs boson at the LHC [1], one of the most crucial task is to unveil the nature of this scalar particle. Current measurement of Higgs couplings in various channels reports certain deviation from the SM expectation, implicating that new physics may exist beyond the TeV energy scale, although there is no direct signature that confirms the existence of new particles. One of the theoretically plausible frameworks for the BSM new physics is the traditional SUSY, which aims to solve the hierarchy problem and propose mechanisms for the electroweak symmetry breaking. In this paper, we are interested to explore the composite Higgs scenario as an alternative option, since it will provide a little Higgs candidate from an underlying strong dynamic sector. There are many varieties of composite Higgs models, with the original one realized in an extra dimensional scenario following the AdS/CFT correspondence [2]. The Higgs potential is calculable in a 5D description of composite Higgs Model, but the particle spectrum in this framework is much more complicated and not easy to make contact with the LHC measurement. Therefore recent effort is more focused on the 4D construction of Composite Higgs theory (CHM) [3, 4, 5]. Since only the lowest lying states are accessible in the future LHC, it is adequate to formulate a predictive description without resorting to an UV completing picture.

One effective approach to qualitatively describe a strong dynamic theory is employing the Callan-Coleman-Wess-Zumino (CCWZ) formalism [6], where the full global symmetry is nonlinearly realized and the Lagrangian is constructed by covariant objects transforming under a local symmetry group. The CCWZ prescription captures the common features in a generic CHM e.g. modified Higgs couplings, while on the other hand it hides the dynamic origin of the partial compositeness and leaves the masses of vector bosons to be less correlated. Here we are going to use the deconstruction method proposed by [4, 5] to parametrize the scenario where one composite Higgs is realized as a pNGB from a spontaneous breaking global symmetry. We are interested in exploring the simplest case, a two-site model with an enlarged global symmetry of  $SO(5)_1 \times SO(5)_2$ . In such a description, only the first level of composite resonances

is available. In contrast to the CCWZ formalism, the symmetry is realized in a linear way via the deconstruction method, with the partial compositeness being manifested as the result of symmetry breaking in the composite sector. In particular non-local terms are possible to be introduced into this two-site model according to the symmetry principle. The existence of non-local terms has crucial impact on the unitarity of  $W_L W_L$  scattering and could change the sign of  $S$  parameter under certain condition. Composite vector-like fermions in  $SO(5)$  representations are necessarily to be incorporated so that the SM fermions will gain the masses. One important motivation for this paper is to explore the influence of composite fermions on the electroweak precision test and estimate their contribution to the Higgs production.

The paper is organized in the following way. We start from a review of the model set up in the two-site description in Section 2. For the gauge bosons, the spectrum is calculated in the unitary gauge and we investigate the influence of non-local term on the  $\pi\pi \rightarrow \pi\pi$  scattering. While for the fermion sector, composite fermions in a basic  $SO(5)$  representation are included and we are going to explore their mixing with the SM fermions. In Section 3, the contribution to  $S$  and  $T$  parameter from the vector and fermion resonances are illustrated in detail, which is further compared with the experimental data by a numerical scanning of the parameter space in this two-site model. Finally in Section 4, we estimate the reduced Higgs production rate from the gluon fusion process by imposing the EW constraints on the composite scale  $f$ .

## 2 two-site model

Let us first review the basic model set up. The two-site model is the simplest scenario to describe a composite Higgs boson, with one site imitating the UV brane and the other site imitating the bulk in a warped extra dimension. Since the symmetry breaking is generalized to be  $SO(5)_1 \times SO(5)_2 / SO(4)$  as depicted in Fig. 1, there are in principle two sets of non-linear sigma fields in order to describe the coset space. One link field  $\Omega_1$  mediating the interaction between the site-1 and the site-2 will break the  $SO(5)_1 \times SO(5)_2$  into a diagonal one  $SO(5)_D$ , while another scalar field  $\Phi_2 = \Omega_2 \phi_0$

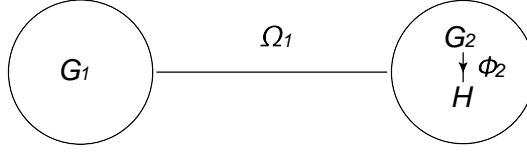


Figure 1: moose diagram for a two-site model, where the global symmetry is  $G_1 \times G_2 = SO(5)_1 \times SO(5)_2$ . The two sites are connected by a link field  $\Omega_1$  and the  $SO(5)_2$  symmetry is spontaneously broken by a local field  $\Phi_2 = \Omega_2 \phi_0$  into a subgroup  $H = SO(4)$ .

located at the site-2 is responsible to break the  $SO(5)_D$  into  $SO(4)$ . Thus we obtain a total of  $10+4$  NGBs. In order to get rid of the redundant NGBs, we are going to gauge a subgroup  $SU(2)_L \times U(1)_Y$  at the site-1 and the full  $SO(5)_2$  symmetry at the site-2. Therefore after the spontaneous symmetry breaking, this two-site model delivers only one copy of Higgs boson.

The two-site model interprets the holographic nature of a composite Higgs arising from a strong dynamics. We would like to illustrate this point by connecting the deconstruction method with the CCWZ formalism. Since elementary fields in the SM will be put at the site-1, they play the roles of the source fields for operators constructed by composite fields residing at the site-2. Let us simply set  $\Omega_2 = 1$  and rescale the kinetic terms to be canonically normalized, thus we obtain an effective Lagrangian equivalent to the CCWZ description. However for the phenomenology relevance in the following discussion, we prefer to investigate the particle spectrum in the unitary gauge, where only the physical degrees of freedom appear in the Lagrangian.

## 2.1 gauge sector

First of all, we need to figure out the gauge interactions with those NGBs. In the unitary gauge, the pion fields in the coset of  $SO(5)/SO(4)$  are parametrized in the sigma fields  $\Omega_1$  and  $\Phi_2$ . As pointed in the Ref. [7], there should be another scalar field  $\Omega_X$ , whose existence is necessary to lift one combination of extra  $U(1)$  gauge fields. Under the nearest gauge interaction principle, the Lagrangian for the gauge bosons

and nonlinear sigma fields is:

$$\begin{aligned}\mathcal{L}_{2-site} &= \frac{f_1^2}{4} \text{Tr} |D_\mu \Omega_1|^2 + \frac{f_2^2}{2} (D_\mu \Phi_2)^T D_\mu \Phi_2 + \frac{f_X^2}{4} |D_\mu \Omega_X|^2 \\ &- \frac{1}{4} \text{Tr} w_{\mu\nu} w^{\mu\nu} - \frac{1}{4} b_{\mu\nu} b^{\mu\nu} - \frac{1}{4} \text{Tr} \rho_{\mu\nu} \rho^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X_{\mu\nu}.\end{aligned}\quad (1)$$

with the covariant derivative terms defined as:

$$\begin{aligned}D_\mu \Omega_1 &= \partial_\mu \Omega_1 - i g_0 w_\mu^a T^{aL} \Omega_1 - i g'_0 b_\mu T^{3R} \Omega_1 + i g_\rho \Omega_1 \rho_\mu^A T^A \\ D_\mu \Omega_X &= \partial_\mu \Omega_X - i g'_0 b_\mu \Omega_X + i g_X X_\mu \Omega_X \\ D_\mu \Phi_2 &= \partial_\mu \Phi_2 - i g_\rho \rho_\mu^A T^A \Phi_2\end{aligned}\quad (2)$$

where  $T^A$  is the generator for  $SO(5)$  group and  $T^{aL}, T^{3R}$  are the generators in the subgroup  $SU(2)_L$  and  $SU(2)_R$ . The broken generator in the coset of  $SO(5)/SO(4)$  is denoted as  $T^{\hat{a}}$ , with  $\hat{a} = 1, 2, 3, 4$ . In the unitary gauge, we will simply set  $\Omega_X = 1$ , thus the remaining sigma fields are parametrized as:

$$\begin{aligned}\Omega_1 &= \exp \left[ i \frac{1}{f} \frac{f_2^2}{f_1^2 + f_2^2} \pi^{\hat{a}} T^{\hat{a}} \right], \\ \Phi_2 &= \exp \left[ i \frac{1}{f} \frac{f_1^2}{f_1^2 + f_2^2} \pi^{\hat{a}} T^{\hat{a}} \right] \phi_0,\end{aligned}\quad (3)$$

with  $\phi_0^t = (0, 0, 0, 0, 1)$  indicating a spontaneous symmetry breaking. Defining the field  $\Phi = \Omega_1 \Phi_2$ , we obtain the physical sigma field for this model set-up:

$$\Phi^T = \frac{1}{\pi} \sin(\pi/f_\pi) (\pi_1, \pi_2, \pi_3, \pi_4, \pi \cot(\pi/f_\pi)) \quad (4)$$

Furthermore the following two-derivative kinetic term is possible to add into the Lagrangian [5], which is a non-local term, but allowed by the symmetries.

$$\begin{aligned}\mathcal{L}_{nl} &= \frac{f_0^2}{2} (D_\mu \Phi)^T D_\mu \Phi \\ D_\mu \Phi &= \partial_\mu \Phi - i g_0 w_\mu^a T^{aL} \Phi - i g'_0 b_\mu T^{3R} \Phi\end{aligned}\quad (5)$$

Since the non-local term contributes to the pion kinetic term, it will modify the pion decay constant. Combining the results from  $\mathcal{L}_{2-site} + \mathcal{L}_{nl}$  and demanding it normalized according to  $\frac{1}{2} (\partial_\mu \pi^{\hat{a}})^2$ , we obtain the following expression for  $f$ :

$$f^2 = f_0^2 + \frac{f_1^2 f_2^2}{f_1^2 + f_2^2}. \quad (6)$$

The particle spectrum for the gauge bosons is easy to be identified before the EWSB, which are mildly corrected after setting  $\langle h \rangle \neq 0$ . For simplicity, we assume that  $f_X = f_1$  and  $g_X = g_\rho$ , the mass eigenstates for those partial composite massive states are:

$$\begin{aligned}\tilde{W}_\mu^{\pm,3} &= \frac{1}{\sqrt{g_0^2 + g_\rho^2}} (g_0 w_\mu^{\pm,3} - g_\rho \rho_{L\mu}^{\pm,3}), \quad B_{1\mu} = \frac{1}{\sqrt{2}} (\rho_{R\mu}^3 - X_\mu) \\ B_{2\mu} &= \frac{1}{\sqrt{4g_0'^2 + 2g_\rho^2}} (g_\rho \rho_{R\mu}^3 + g_\rho X_\mu - 2g_0' b_\mu)\end{aligned}\quad (7)$$

with their masses squared calculated to be:  $m_{\rho_L}^2 = \frac{1}{2}(g_\rho^2 + g_0^2)f_1^2$ ,  $m_{B1}^2 = \frac{1}{2}g_\rho^2 f_1^2$  and  $m_{B2}^2 = \frac{1}{2}(g_\rho^2 + 2g_0'^2)f_1^2$ . There are another six massive gauge bosons, which do not mix with other fields at the leading order approximation. The mass squared for two charged ones  $\rho_{R\mu}^\pm$  is  $m_{\rho_R}^2 = \frac{1}{2}g_\rho^2 f_1^2$ , whereas the mass squared for four axial ones  $a_\mu^{\hat{i}}$ ,  $\hat{i} = 1, 2, 3, 4$  is  $m_a^2 = \frac{1}{2}g_\rho^2(f_1^2 + f_2^2)$ . It should be noticed that with the existence of the non-local term, we can set  $f_2^2 < 0$ , and demanding  $f_1^2 + f_2^2 > 0$  to ensure there are no tachyon modes, which will lead to the condition  $f^2 < f_0^2$  as indicated by Eq. [6].

For the gauge sector, it is worthwhile to investigate whether the unitarity for the pion scattering is partially restored in a two-site framework after the adding of vector resonances. Some works in this direction have already been well done in [8, 9, 10]. We are interested to derive the  $\rho_{L,R}-\pi-\pi$  and  $\pi^4$  vertices which are relevant to the  $\pi\pi \rightarrow \pi\pi$  scattering. From the first term in  $\mathcal{L}_{2-site}$ , we can extract the interaction:

$$\begin{aligned}\mathcal{L}_{\rho\pi^2+\pi^4}^{(1)} &= \frac{(f^2 - f_0^2)^2}{4f^2 f_1^2} g_\rho [\varepsilon^{ijk} \pi^i \partial_\mu \pi^j \rho_{L\mu}^k + (\pi^k \partial_\mu \pi^4 - \pi^4 \partial_\mu \pi^k) \rho_{L\mu}^k] \\ &+ \frac{(f^2 - f_0^2)^2}{4f^2 f_1^2} g_\rho [\varepsilon^{ijk} \pi^i \partial_\mu \pi^j \rho_{R\mu}^k - (\pi^k \partial_\mu \pi^4 - \pi^4 \partial_\mu \pi^k) \rho_{R\mu}^k] \\ &+ \frac{(f^2 - f_0^2)^4}{24f_1^6 f^4} [(\pi^a \partial_\mu \pi^a)^2 - (\pi^a \partial_\mu \pi^b)^2]\end{aligned}\quad (8)$$

and from the second term in  $\mathcal{L}_{2-site}$ , we obtain a similar result:

$$\begin{aligned}\mathcal{L}_{\rho\pi^2+\pi^4}^{(2)} &= \frac{(f^2 - f_0^2)^2}{2f^2 f_2^2} g_\rho [\varepsilon^{ijk} \pi^i \partial_\mu \pi^j \rho_{L\mu}^k + (\pi^k \partial_\mu \pi^4 - \pi^4 \partial_\mu \pi^k) \rho_{L\mu}^k] \\ &+ \frac{(f^2 - f_0^2)^2}{2f^2 f_2^2} g_\rho [\varepsilon^{ijk} \pi^i \partial_\mu \pi^j \rho_{R\mu}^k - (\pi^k \partial_\mu \pi^4 - \pi^4 \partial_\mu \pi^k) \rho_{R\mu}^k] \\ &+ \frac{(f^2 - f_0^2)^4}{6f_2^6 f^4} [(\pi^a \partial_\mu \pi^a)^2 - (\pi^a \partial_\mu \pi^b)^2]\end{aligned}\quad (9)$$

with the index  $i = 1, 2, 3$  and the indexes  $a, b = 1, 2, 3, 4$ . While in the non-local term, there exists additional  $\pi^4$  self interaction term:

$$\mathcal{L}_{\pi^4}^{nl} = \frac{f_0^2}{6f^4} \left[ (\pi^a \partial_\mu \pi^a)^2 - (\pi^a \partial_\mu \pi^b)^2 \right] \quad (10)$$

Following the standard procedure described in [10], we get the partial wave expansion for the pion elastic scattering:

$$\begin{aligned} a_0^0(s)^{(\pi\pi)} = & \frac{1}{16\pi} \left( \frac{(f^2 - f_0^2)^4}{4f^4 f_1^6} + \frac{(f^2 - f_0^2)^4}{f^4 f_2^6} + \frac{f_0^2}{f^4} \right) s \\ & + \frac{g_s^2}{16\pi} \left( \frac{(f^2 - f_0^2)^2}{2f^2 f_1^2} + \frac{(f^2 - f_0^2)^2}{f^2 f_2^2} \right)^2 \left[ \left( \frac{m_\rho^2}{s} + 2 \right) \log \left( \frac{s}{m_\rho^2} + 1 \right) - 1 \right], \quad (11) \end{aligned}$$

with the approximation  $m_\rho = m_{\rho R} \simeq m_{\rho L}$  in the limit  $g_0 \ll g_\rho$ . Here we ignore the width effect from the vector resonance. Depending on the sign choice for the  $f_2^2$ , two distinct scenarios will occur for the unitarity bound. It is noticed from the above equation that when we choose  $f_2^2 > 0$ , the linear and logarithmic divergent terms are both positive, which leads to the result that unless  $f_1$  and  $f_2$  are large enough, the unitarity bound  $|\text{Re} a_0^0(s)^{(\pi\pi)}| \leq \frac{1}{2}$  will be saturated very quickly before the effective cut off scale is approached. However in the other scenario  $f_2^2 < 0$ , one should set the linear divergence to be almost vanishing, so that the high-energy behavior for the pion scattering is mainly determined by the mild logarithmic growing term. In Fig. 2, we plot the unitarity bound in the parameter space  $(f_1, f_2)$  for the two opposite situations by fixing the cut off scale to be a few TeV. It turns out that in the case with  $f_2^2 > 0$ , adding a non-local term is kind of a benefit as it intends to enhance the ratio  $f_1^2/f_2^2$  (preferred by the  $S$  bound) without too much raising the composite scale  $f$ . On the other hand, in the case with  $f_2^2 < 0$ , the partial cancelation in the linear  $s$  term would help restore the perturbative unitarity. But as we should observe from the figure, it largely reduces the unitarity conserving region as compared to the previous case.

## 2.2 fermion sector

In this sector, we discuss the embedding of fermions in the framework of a two-site model. Respecting the full global symmetry, the fermion is supposed to be put into an

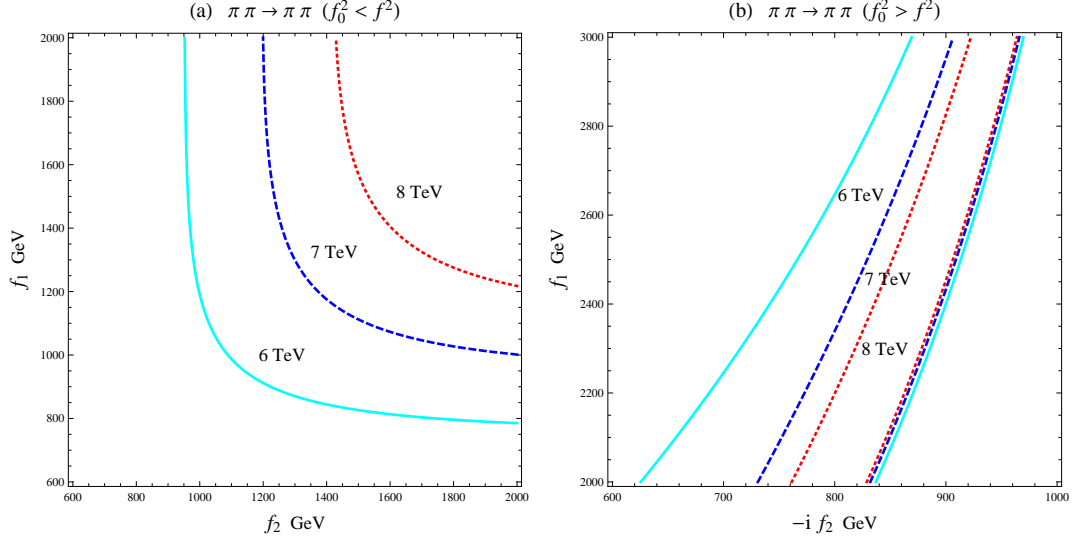


Figure 2: Parameter-space region where the unitarity bound  $|\text{Re}a_0^0(s)^{(\pi\pi)}| \leq \frac{1}{2}$  is violated at energies  $s \leq \Lambda^2$ , for  $\Lambda = 6.0, 7.0, 8.0$  TeV, in the cyan, blue and red lines. we fix the strong coupling to be  $g_\rho = 2.0$ . The left panel is for the case with  $f_2^2 > 0$  and  $f_0 = 800$  GeV, where the region in the right and upper direction is allowed. The right panel is for the case with  $f_2^2 < 0$  and  $f_0 = 1200$  GeV, where the narrow region between the same color lines is allowed.

irreducible  $SO(5)$  representation. Here we are going to focus on the top quark sector and the simplest choice would be the basic representation. In order to let the fermions acquire the right hypercharge assignments, an extra  $U(1)_X$  symmetry is necessary such that quantum numbers are determined by:  $Y = T_{3R} + X$ , and  $Q = T_{3L} + T_{3R} + X$ . Since  $SO(5)$  is spontaneously broken into  $SO(4) \sim SU(2)_L \times SU(2)_R$ , the following decomposition will apply:  $5 = (2, 2) \oplus (1, 1)$ , which gives us one bidoublet  $\psi_{\pm\pm}$  with  $T_{3L}, T_{3R}$  charges  $(\pm\frac{1}{2}, \pm\frac{1}{2})$  and one singlet  $\psi_{00}$ . For the elementary fermions in the site-1, i.e.  $q_L = (t_L, b_L)$ ,  $t_R$  and  $b_R$ , they are embedded into incomplete  $SO(5)$  representations



with the non-dynamic spurion fields being turned off :

$$\xi_L^u = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \end{pmatrix}_{2/3}, \quad \xi_R^u = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ it_R \end{pmatrix}_{2/3} \quad (12)$$

Let us simply introduce one set of composite fermions in the site-2, which should be accommodated in a complete  $SO(5)$  representation:

$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} X_4 + b_4 \\ iX_4 - ib_4 \\ -T_4 + t_4 \\ iT_4 + it_4 \\ \sqrt{2}iT_1 \end{pmatrix}_{2/3} \quad (13)$$

In the above construction we get one doublet  $(t_4, b_4)$ , one non-standard doublet  $(X_4, T_4)$ , where the exotic quark  $X_4$  carries an electric charge of  $5/3$ , and one singlet top  $T_1$ .

Due to the composite nature of our Higgs field, we can not directly couple two SM fermions with one Higgs field. However a bilinear mix interaction is permitted, that is we can use the link field  $\Omega_1$  to connect the SM field in site-1 with the composite field in site-2. It is also possible for us to write down the  $SO(5)$  invariant terms constructed with only the composite fermion fields and pNGBs, so that the SM fermions will gain mass via the partial compositeness. As far as the top quark is concerned, the Lagrangian for the fermion sector is:

$$\begin{aligned} \mathcal{L}_{top} &= \bar{q}_L i \not{D} q_L + t_R i \not{D} t_R + \bar{\psi} i \not{D} \psi \\ &+ c_{tR} \bar{\xi}_R^u \Omega_1 \psi_L + c_{qL} \bar{\xi}_L^u \Omega_1 \psi_R - y_T \bar{\psi}_L \Phi_2 \Phi_2^T \psi_R - m_Y \bar{\psi}_L \psi_R + h.c. \end{aligned} \quad (14)$$

where after the EWSB in the unitary gauge, the  $\Omega_1$  takes the following simple form:

$$\Omega_1 = \begin{pmatrix} I_{3 \times 3} & & \\ & \cos \frac{f}{f_1^2} h & \sin \frac{f}{f_1^2} h \\ & -\sin \frac{f}{f_1^2} h & \cos \frac{f}{f_1^2} h \end{pmatrix}, \quad (15)$$

and the explicit expression for the other scalar field is:

$$\Phi_2^t = \left( 0, 0, 0, \sin \left[ \frac{f_1^2}{f_1^2 + f_2^2} \frac{h}{f} \right], \cos \left[ \frac{f_1^2}{f_1^2 + f_2^2} \frac{h}{f} \right] \right). \quad (16)$$

There are four top quarks with electric charge 2/3 in this two-site model:  $(t, t_4, T_4, T_1)$ , with the relevant mass term:

$$\mathcal{L}_m = \overline{\begin{pmatrix} t_L \\ t_L^4 \\ T_L^4 \\ T_L^1 \end{pmatrix}} M_{top} \begin{pmatrix} t_R \\ t_R^4 \\ T_R^4 \\ T_R^1 \end{pmatrix} + h.c. \quad (17)$$

Since  $v < f_{1,2}, f$ , let us expand the  $M_{top}$  to the order of  $\mathcal{O}(v/f)$ ,

$$M_{top} = \begin{pmatrix} 0 & c_{qL} & 0 & \frac{c_{qL} f_2^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} \\ -\frac{c_{tR} f_2^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} & -m_Y & 0 & -\frac{y_T f_1^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} \\ -\frac{c_{tR} f_2^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} & 0 & -m_Y & -\frac{y_T f_1^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} \\ c_{tR} & -\frac{y_T f_1^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} & -\frac{y_T f_1^2 v}{\sqrt{2} f (f_1^2 + f_2^2)} & -m_Y - y_T \end{pmatrix}. \quad (18)$$

The top quark mass matrix is easy to be analytically diagonalized if we set the Higgs VEV to be zero. From the Lagrangian  $\mathcal{L}_{top}$ , the mass matrix for the two bottom quarks  $(b, b_4)$  can also be extracted. But unlike the top quark case, the bottom quark mass matrix has no dependence on the Higgs field. Since the mixing pattern for the left handed top and bottom quarks coincides with each other at  $\langle h \rangle = 0$ , the following rotation simultaneously transforms them into the mass eigenstates:

$$\begin{aligned} \tilde{t}_L &= \frac{m_Y}{\sqrt{c_{qL}^2 + m_Y^2}} t_L + \frac{c_{qL}}{\sqrt{c_{qL}^2 + m_Y^2}} t_{4L}, \\ \tilde{t}_{4L} &= -\frac{c_{qL}}{\sqrt{c_{qL}^2 + m_Y^2}} t_L + \frac{m_Y}{\sqrt{c_{qL}^2 + m_Y^2}} t_{4L}, \end{aligned} \quad (19)$$

$$\begin{aligned} \tilde{b}_L &= \frac{m_Y}{\sqrt{c_{qL}^2 + m_Y^2}} b_L + \frac{c_{qL}}{\sqrt{c_{qL}^2 + m_Y^2}} b_{4L}, \\ \tilde{b}_{4L} &= -\frac{c_{qL}}{\sqrt{c_{qL}^2 + m_Y^2}} b_L + \frac{m_Y}{\sqrt{c_{qL}^2 + m_Y^2}} b_{4L}. \end{aligned} \quad (20)$$

While for the right handed top quarks, it is the two singlets  $(t_R, T_{1R})$  that would mix with each other and the corresponding rotation is:

$$\begin{aligned}\tilde{t}_R &= \frac{m_Y + y_T}{\sqrt{c_{tR}^2 + (m_Y + y_T)^2}} t_R + \frac{c_{tR}}{\sqrt{c_{tR}^2 + (m_Y + y_T)^2}} t_{1R}, \\ \tilde{t}_{1R} &= -\frac{c_{tR}}{\sqrt{c_{tR}^2 + (m_Y + y_T)^2}} t_R + \frac{m_Y + y_T}{\sqrt{c_{tR}^2 + (m_Y + y_T)^2}} t_{1R}.\end{aligned}\quad (21)$$

Defining the mixing angles  $\sin \theta_L = \frac{c_{qL}}{\sqrt{c_{qL}^2 + m_Y^2}}$  and  $\sin \theta_R = \frac{c_{tR}}{\sqrt{c_{tR}^2 + (m_Y + y_T)^2}}$ , thus to the leading order expansion, the top quark mass can be approximated as,

$$m_{t0} \simeq \frac{|y_T| \sin \theta_L \sin \theta_R v}{\sqrt{2} f}. \quad (22)$$

where  $\sin \theta_L$  and  $\sin \theta_R$  indicate the degree of compositeness for  $t_L$  and  $t_R$  respectively. Notice that though the term proportional to  $y_T$  in Eq. [14] only gives mass to a singlet top  $T_1$ , it is necessary to be present for the SM top  $t_0$  to gain the observed mass. Furthermore, suppose that  $|y_T|$  is of a few TeV energy scale, we need the left handed or the right handed top to mostly origin from the composite sector. The masses for three heavy top quarks are determined by:

$$m_{t4}^2 = c_{qL}^2 + m_Y^2, \quad m_{T4}^2 = m_Y^2, \quad m_{T1}^2 = c_{tR}^2 + (m_Y + y_T)^2 \quad (23)$$

For the composite sector, we generally will set the parameter  $y_T$  to be positive. In such a case, the  $SU(2)_L$  partner for  $T_4$ , i.e. an exotic quark  $X_4$ , would be the lightest fermionic resonance, since it gets no further correction of  $\mathcal{O}(v/f)$  after the EWSB. On the other hand, when we choose a negative  $y_T$ , the lightest fermion could either be the singlet top  $T_1$  or the exotic quark  $X_4$ .

### 3 Electroweak Constraint from $S$ and $T$

Oblique parameters associated with the electroweak precision test puts a severe bound on the parameter space for “universal” models beyond the Standard Model. Let us first recall the definitions of oblique parameters, which are extracted from the two-point functions of weak currents for the gauge bosons.  $S$ ,  $T$  and  $U$  correspond to the residue

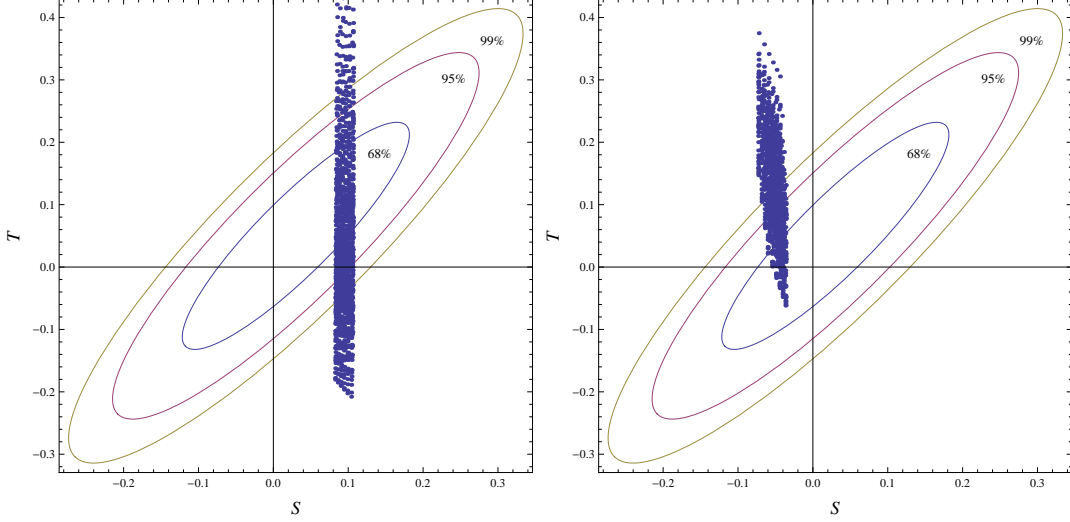


Figure 3: S-T plane for the composite resonance parameter scanning. The ellipses are at the 68% ( $1\sigma$ ), 95% ( $2\sigma$ ) and 99% ( $3\sigma$ ) confidence levels. The strong coupling is fixed to be  $g_\rho = 2.0$ . In the left contour parameters are in the range (GeV):  $3000 < c_{tR}, y_T < 3200$ ,  $3000 < m_Y, f_1 < 3500$ ,  $800 < f_2 < 830$  and  $f_0 = 0.0$ ; In the right contour parameters are in the range (GeV):  $3400 < c_{tR} < 3600$ ,  $3000 < y_T < 3200$ ,  $3000 < m_Y, f_1 < 3500$ ,  $800 < -if_2 < 830$  and  $f_0 = 1200$ . The remaining parameter  $c_{qL}$  is calculated with an input top quark mass and the points which are located in the ellipses pass the EWPT.

coefficients for expansion up to the order of  $p^2$  after fixing gauge couplings, Higgs VEV and imposing the  $U(1)_{em}$  gauge invariance [11, 12]. Roughly speaking, heavy fields from the EW symmetry breaking sector additively contribute to the  $S$ , while the effect of isospin breaking is counted by the  $T$  and  $U$ . In the two-site composite Higgs model, there is a tree level mixing between the elementary gauge fields and composite gauge fields. Thus after integrating out heavy spin-1 resonances (both the vector and axial bosons), we find the dominating contribution to  $S$  parameter is:

$$\begin{aligned} \Delta S &= -16\pi \cdot \Pi'_{W^3B}(0) \\ &= \frac{8\pi}{g_\rho^2} \left[ 1 - \frac{f_1^4}{(f_1^2 + f_2^2)^2} \right] \frac{v^2}{f^2} \end{aligned} \quad (24)$$

As we should notice that although this effect is proportional to  $v^2/f^2$ , the tree level deviation still imposes a stringent constraint due to a factor of  $8\pi$  unless the strong

coupling  $1 < g_\rho < 4\pi$  is large enough. It is argued in ref. [13] that the  $S$  parameter is generally positive in most extra dimension scenarios. However, in the two-site model with the existence of non-local term, it is possible to enforce the condition  $0 < f_1^2 + f_2^2 < f_1^2$ , thus we obtain a negative  $S$  parameter. Moreover we have the freedom to ensure  $|f_2| \ll |f_1|$ , therefore the leading order contribution to the  $S$  parameter is further suppressed, with the magnitude of its deviation being kept in the vicinity of 0.1.

Since those composite vector resonances remain in the irreducible  $SO(4) \simeq SU(2)_L \times SU(2)_R$  representations after the spontaneous global symmetry breaking  $SO(5) \rightarrow SO(4)$ , the tree level deviation to  $T$  and  $U$  is zero due to the custodial symmetry protection. Nonetheless the correction will arise at the loop level.

$$\Delta T = \Delta U = 0 \quad (25)$$

Another calculable source for  $S$  and  $T$  origins from the reduced gauge couplings with Higgs boson, which give rise to the infrared (IR) contribution. It is convenient to derive those couplings in the holographic basis by expanding the Higgs field around its VEV,

$$\begin{aligned} & \frac{1}{4} g^2 f^2 W_\mu^+ W_\mu^- \sin^2(h/f) + \frac{1}{8} (g^2 + g'^2) f^2 Z_\mu Z_\mu \sin^2(h/f) \\ \Rightarrow & \frac{1}{2} \frac{2m_W^2}{v} (v + a h) W_\mu^+ W_\mu^- + \frac{1}{2} \frac{m_Z^2}{v} (v + a h) Z_\mu Z_\mu + \mathcal{O}(h^2). \end{aligned} \quad (26)$$

with the masses of gauge bosons and the parameter  $|a| < 1$  determined by:

$$m_W^2 = \frac{g^2 f^2 \sin^2(v/f)}{4}, \quad m_Z^2 = \frac{(g^2 + g'^2) f^2 \sin^2(v/f)}{4}, \quad a = \cos(v/f) \quad (27)$$

In the SM, there is an exact cancelation of logarithmic divergence for the  $S$  and  $T$ , which is spoiled by the reduced Higgs couplings with gauge bosons. Therefore the IR contribution in fact describes a running effect from the EW scale till the composite scale in the effective theory, where by the NDA estimation  $\Lambda_{NDA} = 4\pi f$ .

$$\Delta S_{IR} = \frac{1}{6\pi} \left[ \sin^2(v/f) \log \left( \frac{\Lambda}{m_h} \right) + \log \left( \frac{m_h}{m_{h,ref}} \right) \right], \quad (28)$$

$$\Delta T_{IR} = -\frac{3}{8\pi c_w^2} \left[ \sin^2(v/f) \log \left( \frac{\Lambda}{m_h} \right) + \log \left( \frac{m_h}{m_{h,ref}} \right) \right] \quad (29)$$

Under the condition that  $f \sim 1.0$  TeV, i.e. assuming no large splitting exists between the Higgs VEV and the composite scale  $f$ , the IR contribution generally constitutes a sizable portion to both  $S$  and  $T$ .

Finally let us discuss more about the fermion loop correction to EW precision test. The evaluation for the  $S$  and  $T$  from various types of vector-like quarks is given in the reference [14]. Some detail studies of  $T$  parameter constraint on vector-like singlet, doublet and triplet quarks could also be found in [15, 16]. I will give the analytic expressions of  $S$  parameter from vector-like quark loops, especially for the nonstandard doublet scenario in the appendix. It is worth to point out that since the exotic quark  $X_4$  carries an electric charge of  $\frac{5}{3}$ , the function  $\psi_+$  defined in the original work [14] will be modified. For the  $S$  parameter, the virtual fermion effect is generally subleading, but not totally irrelevant, which leads to the consequence that the  $S$  is less model dependent on the mixing parameters. By contrast the fermion contribution to the  $T$  is more important and needs to take into consideration. We expect that the  $T$  obtains a substantial positive shift through quark mixings, so that it will partially compensate the negative IR correction. The magnitude of  $T_{ferm}$  in a generic composite Higgs model can be estimated in the limit  $|f_2| \ll |f_1|$  as:

$$\Delta T_{ferm} \sim \frac{N_c}{4\pi s_W^2 g^2} \frac{y_T^4 \sin^4 \theta_{L,R}}{M_T^2 f^2} \sin \frac{v^2}{f^2}, \quad (30)$$

where  $M_T$  collectively stands for the mass of a composite vector-like quark. In order to get a quantitative understanding, we could assume that  $\sin \theta_{L,R} \sim 0.6$ ,  $M_T \sim |y_T| \sim 3.0$  TeV, and  $f \sim 1.0$  TeV, thus an estimation is obtained  $\Delta T_{ferm} \sim 0.17$ , which is numerically competitive with the IR correction. The sign for the  $T$  parameter is determined by the isospin of the heavy top quark. As we have shown in the mass matrix for top-like quarks, the mixing is entangled with each other. For simplicity, we consider that the top quark separately mixes with one type of vector-like quark each time. Using the equations presented in [16], it is possible to exactly evaluate the  $T$  parameter for various scenarios. For the SM top  $t_0$  mixing with one singlet  $T_1$ , or mixing with one doublet  $(t_4, b_4)$ ,  $\Delta T_{ferm}$  is always positive. The situation becomes opposite for the nonstandard doublet  $(X_4, T_4)$ , as its modification to the  $W_3^\mu W_3^\mu$  form

factor is bigger than the other two cases. In the small mixing limit, the last type mixing gives a negative contribution to the  $T$  parameter. In this two-site model, we can find that because the  $t_0$  mixing with  $T_4$  is larger than with  $t_4$ , under the condition  $M_{T4} < M_{t4}$ , the negative contribution from nonstandard doublet will overcome the positive one from the doublet. However provided that additional positive contribution obtained from the mixing with  $T_1$  is large enough, the combining result  $\Delta T_{ferm}$  would still be positive, as we can prove this point using the numerical scanning.

In Fig. 3, we show the numerical results for  $S$  and  $T$  by scanning the parameter set  $(c_{qL}, c_{tR}, m_Y, y_T, f_0, f_1, f_2)$  in typical ranges. It is observed that the  $S$  is mainly sensitive to  $f_0$  and  $f_{1,2}$ , whereas the  $T$  is more dependent on the mixing parameters for the fermions. We are intending to find out viable parameter regions which are compatible with experimental values, i.e.  $S = 0.03 \pm 0.10$  and  $T = 0.05 \pm 0.12$  with a correlation coefficient of  $\rho_{col} = 0.89$ . In the left contour, we illustrates one case with no non-local term, i.e.  $f_0 = 0$ , therefore the  $S$  parameter is bound to be positive due to the condition  $f_1^2 < f_1^2 + f_2^2$ . Combining all the mixing effects from the SM chiral tops with composite vector-like quarks, we find out that there is enough possibility for the  $T$  to be shifted into the positive region. While in the right contour, another case with a non-local term is present. Since we enforce an opposite condition  $f_1^2 > f_1^2 + f_2^2$ , the sign of  $S$  parameter is tuned to be negative. In the latter case, the points with a negative  $T$  are more consistent with the electroweak data.

## 4 Gluon fusion to Higgs Production

Since the Yukawa couplings for the composite fermions are crucial for the gluon fusion process, we are going to briefly comment their contribution to the Higgs production. For all the fermion fields in the mass eigenstates, let us assume that they are interacting with the Higgs field in the following way:

$$\mathcal{L}_h = \sum M_i \bar{\psi}_i \psi_i + \sum Y_{ii} h^0 \bar{\psi}_i \psi_i \quad (31)$$

where  $M_i$  and  $Y_{ii}$  are the mass and Yukawa coupling for each fermion. Notice that

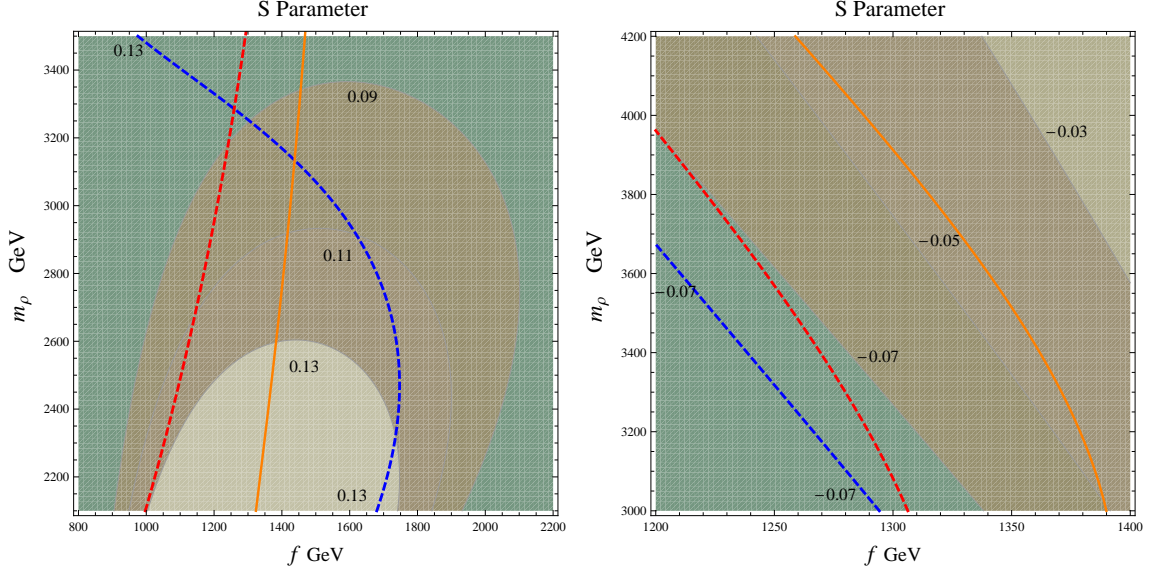


Figure 4: S parameter contour and unitarity bound with  $\Lambda = 8$  TeV on the plane of  $(m_\rho, f)$ . In the left contour, we set  $f_0 = 800$  GeV and demand that  $f_1^2 + f_2^2 > f_1^2$ . The orange line corresponds to the unitarity bound, where the region in the right direction is allowed. The  $S = +0.13$  bound (blue dashed line) and the unitarity bound (red dashed line) at  $f_0 = 0.0$  GeV is shown for comparison. While in the right contour, we set  $f_0 = 1600$  GeV and instead enforce  $f_1^2 + f_2^2 < f_1^2$ . The orange line represents the unitarity bound, where the region in the lower direction is allowed. The  $S = -0.07$  bound (blue dashed line) and the unitarity bound (red dashed line) at  $f_0 = 1550$  GeV is shown for comparison.

in our simplified model, only the top-like quarks have interaction with the Higgs field. The production rate for a Higgs boson from fermion loops is proportional to:

$$\sigma(gg \rightarrow h^0) \propto \left| \sum_i \frac{Y_{ii}}{M_i} A_{1/2}(\tau_i) \right|^2, \quad \tau_i = \frac{m_h^2}{4M_i^2} \quad (32)$$

For the case of a light Higgs boson, we generally have  $M_{Q,t} \gg m_h \gg m_b$ , with the index  $Q$  stands for the heavy vector-like quarks. In order to make contact with the low energy phenomenology, one just needs to take advantage of the approximation for  $A_{1/2}(\tau_i)$  in certain limits, i.e.  $A_{1/2}(\tau) \rightarrow 4/3$ , for  $\tau \rightarrow 0$ , and  $A_{1/2}(\tau) \rightarrow 0$ , for  $\tau \rightarrow \infty$ .



Furthermore relating the sum of  $\sum y_{ii}/M_i$  to the determinant of the mass matrix:

$$\sum_i \frac{Y_{ii}}{M_i} = \frac{\partial \log(\det M)}{\partial v}, \quad (33)$$

we can effectively evaluate the production rate without rotating into the mass eigenstate. Since the  $X_{5/3}$  and  $b_4$  in this model do not couple to the Higgs field, by neglecting the bottom quark contribution we obtain a concise result :

$$\frac{\sigma(gg \rightarrow h^0)_{CHM}}{\sigma(gg \rightarrow h^0)_{SM}} \simeq \frac{2v}{f} \cot\left(\frac{2v}{f}\right) = 1 - \frac{4}{3} \frac{v^2}{f^2} \quad (34)$$

Therefore in the case with only one multiplet of composite quarks, the gluon fusion production of Higgs boson is generally reduced with respect to the SM scenario, as pointed out by the pioneer work [17]. However it is noticed in ref. [18] that through introducing the  $h$  dependent bottom quark mixing with composite quarks, an enhanced  $h^0 gg$  coupling is possible to be realized in the composite Higgs scenario.

Through constraining the decay constant using the unitarity and EWPT, we will be able to estimate the reduced percentage in this model. The  $S$  parameter, especially its dominate part  $\Delta S_{fit} \simeq \Delta S_{tree} + \Delta S_{IR}$ , imposes a stringent bound on  $m_\rho$  and  $v^2/f^2$  by requiring that  $-0.07 < S < 0.13$ . On the other hand, we should also require the unitarity bound to be conserved till a relative large effective cut off scale, e.g. 8 TeV. The Fig. 4 interprets the  $S$  parameter and unitarity bound in the  $(m_\rho, f)$  plane. For the case  $f_1^2 + f_2^2 > f_1^2$ , it turns out that a larger  $f_0$  leads to a growing  $f$ , but the mass scale  $m_\rho$  will be lowered, resulting in more parameter region compatible with the experimental data. However, for the opposite case  $f_1^2 + f_2^2 < f_1^2$ , the unitarity bound intends to push  $f$  to be smaller, while the  $S$  parameter requires a larger  $f$ . Thus one has to increase the  $f_0$  in order to gain more compatible region. Under either situation, we will find out that after imposing necessary constraints, an upper bound  $v^2/f^2 < 0.042$  is generally permitted, which in turn translates into a rough estimation  $\sigma_{CHM}/\sigma_{SM} > 0.94$  for the reduced Higgs production rate. Notice that a lower production rate is applicable if one reduces the  $\Lambda_{eff}$  for the unitarity bound.

## 5 Conclusion

In this paper, we study the minimal  $SO(5)/SO(4)$  Composite Higgs Model in a two-site scenario, where only the first level of composite resonances is present in comparison with the KK modes in an extra dimension theory. In addition to the nearest neighbor interaction, we especially investigate the effects of non-local term on the perturbative unitarity and the EW precision test. In the scenario  $f_2^2 > 0$ , the existence of non-local term will lead to a lower bound for the vector resonance mass  $m_\rho$ , which ameliorates the compatibility of this two-site model with experimental data. On the other hand, the non-local term is necessary to be added in the scenario  $f_2^2 < 0$ . Under such a situation, there is a tension between the unitarity requirement and the negative  $S$  parameter bound. Thus we should increase the  $f_0$  in order to relieve this tension and achieve a relative large effective cut off scale  $\Lambda_{eff}$ .

For simplicity, vector-like composite fermions are embedded in the  $SO(5)$  basic representations, which will mix the SM quarks via bilinear interactions. Under the situation of a positive  $S$  parameter, we prefer a positive  $T$  for a better fit with the EW precision test. Through the parameter scanning, we have shown that there is enough possibility that the positive correction to the  $T$  from vector-like fermions could dominate the negative IR contribution from the reduced Higgs couplings. Furthermore, the effects of vector-like fermions on the gluon fusion Higgs production is discussed. It turns out the production rate will at most be reduced around 6% provided that we demand a strict unitarity and EWPT to be satisfied.

## Acknowledgments

H.Cai is supported by the postdoctoral foundation under the Grant No. 2012M510001, and in part supported by National Nature Science Foundations of China (NSFC) under the Contract No. 10925522.

## Appendix

In this appendix, we are going to collect the contribution to  $S$  parameter from vector-like fermions in singlet, doublet and nonstandard doublet scenarios. Consider the top quark mixing with each type of vector-like fermion in the following way:

$$\begin{aligned}\mathcal{L}_{top} \supset & -m_t \bar{t}_L t_R - x_{T_1} \bar{t}_L T_{1R} - x_{t_4} \bar{t}_R t_{4L} - x_{T_4} \bar{t}_R T_{4L} \\ & - M_{T_1} \bar{T}_1 T_1 - M_{t_4} \bar{t}_4 t_4 - M_{T_4} \bar{T}_4 T_4 + h.c.\end{aligned}\quad (35)$$

For a singlet vector-like fermion, the mixing angles are determined by diagonalizing the mass matrix,

$$\sin \theta_u^L = \frac{M_{T_1} x_{T_1}}{\sqrt{(M_{T_1}^2 - m_t^2)^2 + M_{T_1}^2 x_{T_1}^2}}, \quad \sin \theta_u^R = \frac{x_{T_1}}{M_{T_1}} \sin \theta_u^L. \quad (36)$$

While for a doublet or nonstandard doublet, the L.H. and R.H. mixing angles are exchanged with respect to the singlet scenario,

$$\sin \theta_u^R = \frac{M_{t_4(T_4)} x_{t_4(T_4)}}{\sqrt{(M_{t_4(T_4)}^2 - m_t^2)^2 + M_{t_4(T_4)}^2 x_{t_4(T_4)}^2}}, \quad \sin \theta_u^L = \frac{x_{t_4(T_4)}}{M_{t_4(T_4)}} \sin \theta_u^R. \quad (37)$$

In terms of mixing angles, the  $S$  parameter for each kind of scenario can be expressed by the following equations:

$$\begin{aligned}\Delta S_{t_0-T_1} &= \frac{3}{2\pi} [\sin^2 \theta_u^L \psi_+(y_{T_1}, y_b) - \sin^2 \theta_u^L \psi_+(y_t, y_b) \\ &\quad - \cos^2 \theta_u^L \sin^2 \theta_u^L \chi_+(y_{T_1}, y_t)] \\ \Delta S_{t_0-t_4} &= \frac{3}{2\pi} [\sin^2 \theta_u^L \psi_+(y_{t_4}, y_b) - \sin^2 \theta_u^L \psi_+(y_t, y_b) \\ &\quad + (\cos^2 \theta_u^L + \cos^2 \theta_u^R) \psi_+(y_{t_4}, y_{b_4}) + (\sin^2 \theta_u^L + \sin^2 \theta_u^R) \psi_+(y_t, y_{b_4}) \\ &\quad + 2 \cos \theta_u^L \cos \theta_u^R \psi_-(y_{t_4}, y_{b_4}) + 2 \sin \theta_u^L \sin \theta_u^R \psi_-(y_t, y_{b_4}) \\ &\quad - \cos^2 \theta_u^R \sin^2 \theta_u^R \chi_+(y_t, y_{t_4})] \\ \Delta S_{t_0-T_4} &= \frac{3}{2\pi} [\sin^2 \theta_u^L \psi_+(y_{T_4}, y_b) - \sin^2 \theta_u^L \psi_+(y_t, y_b) \\ &\quad + (\sin^2 \theta_u^L + \sin^2 \theta_u^R) \psi_+(y_{X_4}, y_t) + (\cos^2 \theta_u^L + \cos^2 \theta_u^R) \psi_+(y_{X_4}, y_{T_4}) \\ &\quad + 2 \sin \theta_u^L \sin \theta_u^R \psi_-(y_{X_4}, y_t) + 2 \cos \theta_u^L \cos \theta_u^R \psi_-(y_{X_4}, y_{T_4}) \\ &\quad - (4 \cos^2 \theta_u^L \sin^2 \theta_u^L + \cos^2 \theta_u^R \sin^2 \theta_u^R) \chi_+(y_t, y_{T_4}) \\ &\quad - 4 \cos \theta_u^L \sin \theta_u^L \cos \theta_u^R \sin \theta_u^R \chi_-(y_t, y_{T_4})]\end{aligned}\quad (38)$$

where the rescaled mass squared is defined as  $y_i = M_i^2/m_Z^2$ , with  $M_i$  representing the mass of vector-like quark and the functions of  $\chi_+$ ,  $\chi_-$ ,  $\psi_+$  and  $\psi_-$  are:

$$\begin{aligned}\chi_+(y_1, y_2) &= \frac{5(y_1^2 + y_2^2) - 22y_1y_2}{9(y_1 - y_2)^2} + \frac{3y_1y_2(y_1 + y_2) - y_1^3 - y_2^3}{3(y_1 - y_2)^3} \ln \frac{y_1}{y_2}, \\ \chi_-(y_1, y_2) &= -\sqrt{y_1y_2} \left( \frac{y_1 + y_2}{6y_1y_2} - \frac{y_1 + y_2}{(y_1 - y_2)^2} + \frac{2y_1y_2}{(y_1 - y_2)^3} \ln \frac{y_1}{y_2} \right), \\ \psi_+(y_\alpha, y_i) &= \frac{1}{3} - \frac{1}{3}(Q_\alpha + Q_i) \ln \frac{y_\alpha}{y_i}, \quad \psi_-(y_\alpha, y_i) = -\frac{y_\alpha + y_i}{6\sqrt{y_\alpha y_i}}.\end{aligned}\tag{39}$$

where our function  $\psi_+(y_\alpha, y_i)$ , with the index  $\alpha$  for an up-type quark and the index  $i$  for a down-type quark, is in fact dependent on the sum of electric charges  $Q_\alpha + Q_i$ , which generalizes the previous result reported in the Ref. [14]. Notice that for the mixing of  $t_0$  with  $T_4$ , the argument  $y_{X4}$  should be put in front of the argument  $y_{T4}$  or  $y_t$ , due to the opposite isospin assignment in a non-standard doublet.

## References

- [1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B **716** (2012) 1 arXiv:1207.7214 [hep-ex]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B **716** (2012) 30 arXiv:1207.7235 [hep-ex].
- [2] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671**, 148 (2003) [hep-ph/0306259]; K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719**, 165 (2005) [hep-ph/0412089].
- [3] C. Anastasiou, E. Furlan and J. Santiago, Phys. Rev. D **79**, 075003 (2009) [arXiv:0901.2117 [hep-ph]].
- [4] G. Panico and A. Wulzer, JHEP **1109**, 135 (2011) arXiv:1106.2719 [hep-ph];
- [5] S. De Curtis, M. Redi and A. Tesi, JHEP **1204**, 042 (2012) [arXiv:1110.1613 [hep-ph]].
- [6] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan Jr., S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2247 (1969).

- [7] M. Carena, L. Da Rold and E. Ponton, arXiv:1402.2987 [hep-ph].
- [8] R. Contino, D. Marzocca, D. Pappadopulo and R. Rattazzi, JHEP **1110**, 081 (2011) [arXiv:1109.1570 [hep-ph]].
- [9] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra and J. Terning, JHEP **1211**, 003 (2012) [arXiv:1205.4032 [hep-ph]].
- [10] H. Cai, Phys. Rev. D **88**, no. 3, 035018 (2013) [arXiv:1303.3833 [hep-ph]].
- [11] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992);
- [12] R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B **703**, 127 (2004) [hep-ph/0405040].
- [13] K. Agashe, C. Csaki, C. Grojean and M. Reece, JHEP **0712**, 003 (2007) [arXiv:0704.1821 [hep-ph]].
- [14] L. Lavoura and J. P. Silva, Phys. Rev. D **47** (1993) 2046;
- [15] G. Cacciapaglia, A. Deandrea, D. Harada and Y. Okada, JHEP **1011**, 159 (2010) [arXiv:1007.2933 [hep-ph]].
- [16] H. Cai, JHEP **1302**, 104 (2013) arXiv:1210.5200 [hep-ph].
- [17] A. Falkowski, Phys. Rev. D **77**, 055018 (2008) [arXiv:0711.0828 [hep-ph]].
- [18] A. Azatov and J. Galloway, Phys. Rev. D **85**, 055013 (2012) [arXiv:1110.5646 [hep-ph]].